Lecture 19 - March 24

Model Checking

Temporal Operators: U, W, R
Formulating English Sentences in LTL

Announcements/Reminders

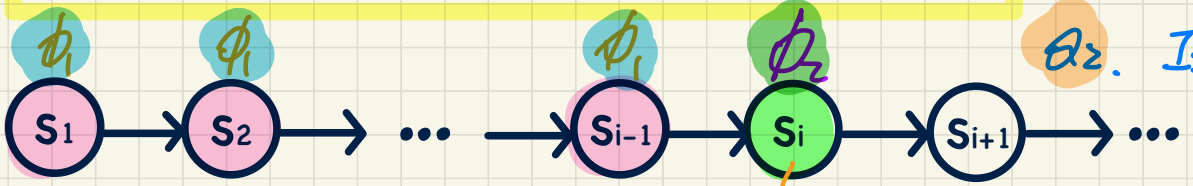
Wed
↳ 20 min
↳ tips.

- **WrittenTest2** this Thursday
- **Lab4** to be released
- Office Hour this week: 3pm on Wed, Thu
- TA contact information (on-demand for labs) on eClass

Path Satisfaction: Temporal Operations (4)

$\pi \models \phi_1 \, U \, \phi_2$ (until)

There is some future state satisfies ϕ_2, and until then, all states satisfy ϕ_1 .



whether ϕ_1 is true here not, does not matter.

Formulation (over a path)

$$\pi \models \phi_1 \, U \, \phi_2 \iff \left(\exists i \cdot i \geq 1 \wedge \left(\pi^i \models \phi_2 \right) \wedge \left(\forall j \cdot 1 \leq j \leq i-1 \Rightarrow \pi^j \models \phi_1 \right) \right)$$

ϕ_1 ← LTL formula

$\pi^i \models \phi_2 \rightarrow F\phi_2$

Q1. Is it ok that $G\phi_1$ but $G\neg\phi_2$?

$G\neg\phi_2 \rightarrow \neg F\phi_2$

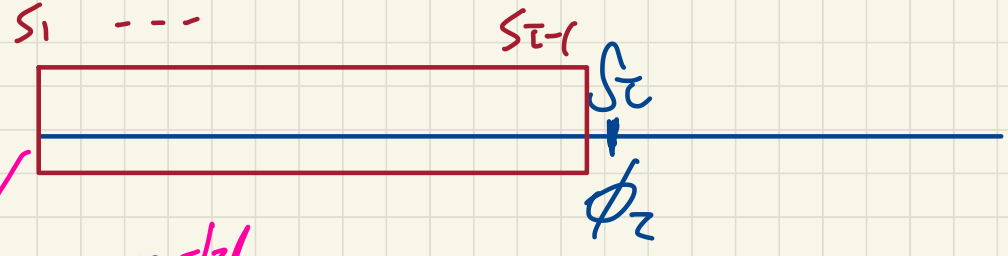
No. $\rightarrow \phi_1 \, U \, \phi_2 \rightarrow$ false

Q2. Is it ok that $G\phi_1$ and $F\phi_2$?

True $\rightarrow \phi_1 \, U \, \phi_2 \rightarrow$ true

$\phi_1 \; \phi_1 \; \phi_1 \; \phi_1 \; \ldots \; \phi_2$

Given a path : $F\phi_1 \wedge F\phi_2$



not necessarily
$G\phi_1$ from s_1 to s_{i-1},
given only $F\phi_1$.

$\pi \vDash \phi_1 \,\mathcal{U}\, \phi_2 \iff \left(\exists i \cdot i \geq 1 \wedge \left(\pi^i \vDash \phi_2 \wedge \left(\forall j \cdot 1 \leq j \leq i-1 \Rightarrow \pi^j \vDash \phi_1 \right) \right) \right)$

ϕ_1 < TK < formula

$\exists i \cdot i \geq 1 \wedge \left(\pi^{\textcircled{i}} \vDash \phi_2 \wedge \left(\forall j \cdot 1 \leq j \leq i-1 \Rightarrow \pi^j \vDash \phi_1 \right) \right)$

$i = 1$

$1 \leq j \leq 0$

$\forall j \cdot \text{false} \Rightarrow \pi^j \vDash \phi_1$

true

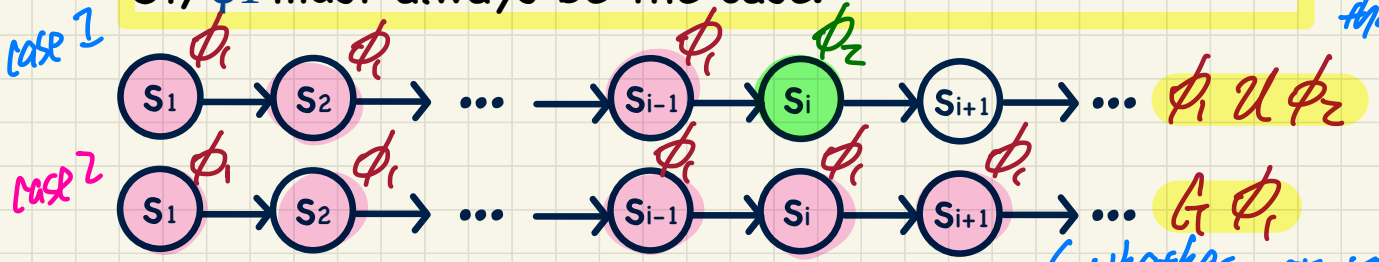
Path Satisfaction: Temporal Operations (5)

weak until

$\pi \models \phi_1 \, W \, \phi_2$

If there is ever a future state that satisfies ϕ_2, then until then, all states satisfy ϕ_1.

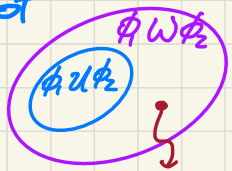
Or, ϕ_1 must always be the case.



Q1. Is it ok that $G \, \phi_1$ but $G \neg \phi_2$?
ok → $\phi_1 \, W \, \phi_2$

Q2. Is it ok that $G \, \phi_1$ and $F \, \phi_2$? (TMP.)
↳ ok.

(whether or not $F \, \phi_2$ does not matter)



Formulation (over a path)

$\phi_1 \, W \, \phi_2 \iff \phi_1 \, U \, \phi_2$

case 1

case 2 $G \, \phi_1$

$(\forall k \cdot k \geq 1 \Rightarrow \pi^k \models \phi_1)$

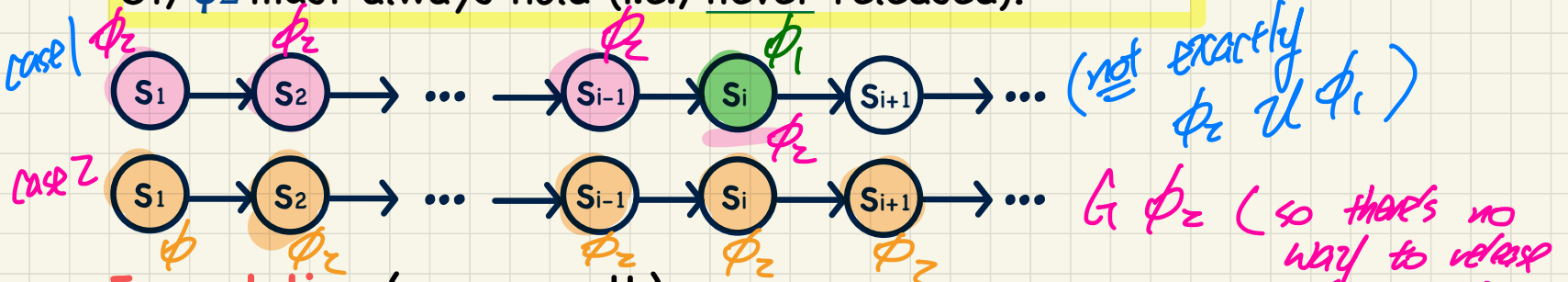
Path Satisfaction: Temporal Operations (6)

release

$\pi \models \phi 1 \; R \; \phi 2$

If there is ever a future state that satisfies $\phi 1$, then until then, all states satisfy $\phi 2$.

Or, $\phi 2$ must always hold (i.e., never released).



Formulation (over a path)

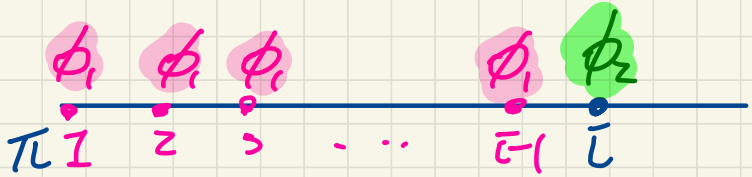
$$\pi \models \phi_1 \; R \; \phi_2 \iff \left(\exists i \cdot i \geq 1 \wedge \left(\pi^i \models \phi_1 \wedge \left(\forall j \cdot j \leq i \cdot \pi^j \models \phi_2\right)\right)\right) \vee \left(\forall k \cdot k \geq 1 \Rightarrow \pi^k \models \phi_2\right)$$

case 1 / case 2 / $G \; \phi_2$

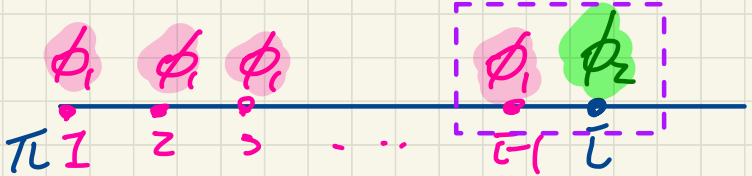
contrast: $\pi \models \phi_2 \; \mathcal{U} \; \phi_1$

U, W, R

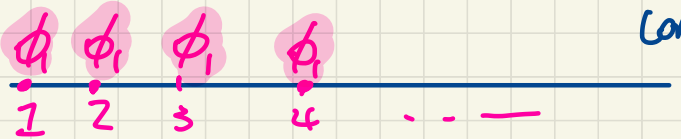
Until $\phi_1 \, U \, \phi_2$



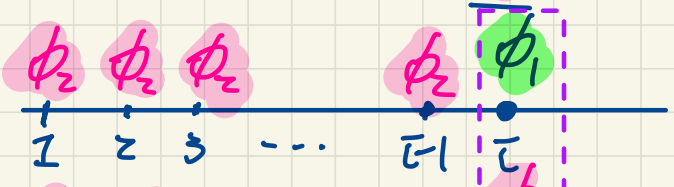
Weak Until $\phi_1 \, W \, \phi_2$



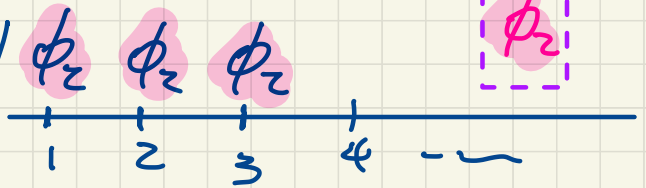
(or) V



Release $\phi_1 \, R \, \phi_2$



(or) V



Formulating Natural Language in LTL (1)

Natural Language:

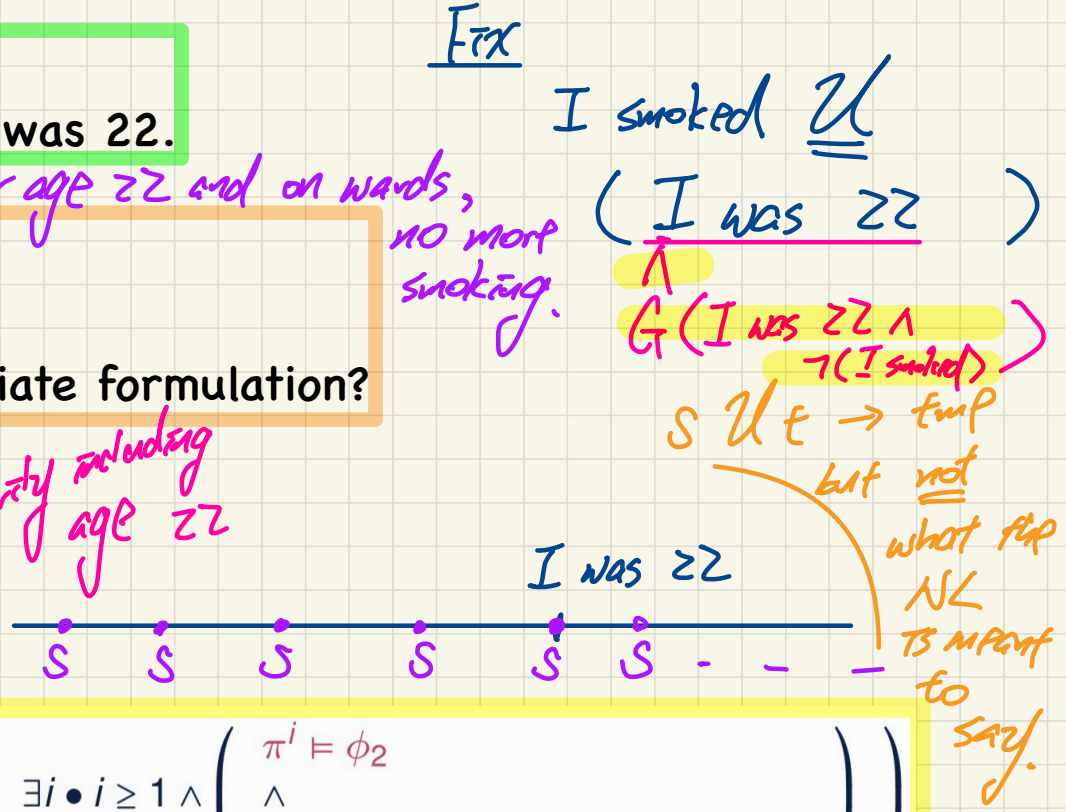
I had smoked until I was 22.

Atom t: I was 22

Atom s: I smoke

Q. Is s U t an appropriate formulation?

$$\pi \models \phi_1 \mathbf{U} \phi_2 \iff \left(\exists i \bullet i \geq 1 \wedge \left(\begin{array}{l} \pi^i \models \phi_2 \\ \wedge \\ (\forall j \bullet 1 \leq j \leq i-1 \Rightarrow \pi^j \models \phi_1) \end{array} \right) \right)$$



Formulating Natural Language in LTL (2.1)

Natural Language:

It's impossible to reach a state where the system is started but not ready.

$G\,\phi \equiv \neg F \neg \phi$

$F\,\phi \equiv \neg G \neg \phi$

Assumed atoms:

- started
- ready

$\neg (F (started \wedge \neg ready))$

LTL Formulation

$\neg (F (started \wedge \neg ready))$

exercise?

$\equiv G (\neg (started \wedge \neg ready))$

$\equiv G (\neg started \vee ready)$ $started \Rightarrow ready$

Formulating Natural Language in LTL (2.2)

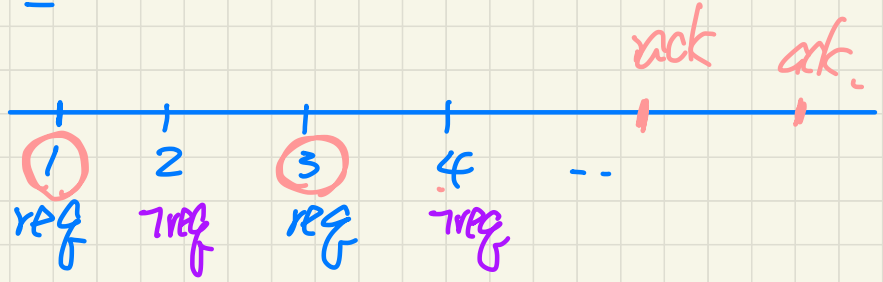
Natural Language:

Whenever a request is made, (G)
it will be acknowledged eventually. (F)

Assumed atoms:

- requested
- acknowledged

$$\underline{G}\ (\text{requested} \Rightarrow F\,\text{ack.})$$



LTL Formulation

exercise.

Formulating Natural Language in LTL (2.3)

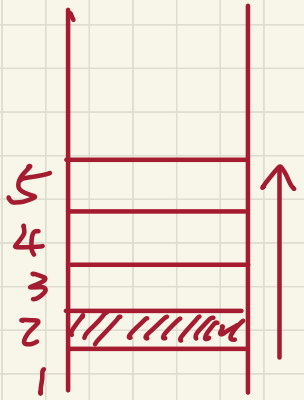
Natural Language:

An elevator traveling upwards at the 2nd floor
does not change its direction
when it has passengers wishing to to to the 5th floor.

Assumed atoms:

- floor2, floor5
- directionUp
- buttonPressed5

LTL Formulation



$$G\left(floor2 \wedge buttonPressed5 \Rightarrow X(G\ directionUp)\right)$$

$$(directionUp\ \mathcal{U}\ floor5)$$

↓ W ?